EJERCICIOS LOGARITMOS. SOLUCIONES

1.- Calcula, aplicando la definición, los siguientes logaritmos:

a) $\log_3 27 = y \Leftrightarrow 3^y = 27 \Leftrightarrow 3^y = 3^3 \Leftrightarrow y = 3$

Por tanto, $\log_3 27 = 3$

b) $\log_{\frac{1}{2}} 64 = y \Leftrightarrow \left(\frac{1}{2}\right)^y = 64 \Leftrightarrow 2^{-y} = 2^6 \Leftrightarrow -y = 6 \Leftrightarrow y = -6$

Por tanto, $\log_{\frac{1}{2}} 64 = -6$

c) $\log_2 128 = y \Leftrightarrow 2^y = 128 \Leftrightarrow 2^y = 2^7 \Leftrightarrow y = 7$

Por tanto, $\log_2 128 = 7$

d) $\log_{\sqrt{2}} 32 = y \Leftrightarrow (\sqrt{2})^y = 32 \Leftrightarrow \left(2^{\frac{1}{2}}\right)^y = 2^5 \Leftrightarrow 2^{\frac{y}{2}} = 2^5 \Leftrightarrow \frac{y}{2} = 5 \Leftrightarrow y = 10$

Por tanto, $\log_{\sqrt{2}} 32 = 10$

e) $\log_{\frac{1}{3}} \sqrt[3]{9} = y \Leftrightarrow \left(\frac{1}{3}\right)^y = \sqrt[3]{9} \Leftrightarrow 3^{-y} = \sqrt[3]{3^2} \Leftrightarrow 3^{-y} = 3^{\frac{2}{3}} \Leftrightarrow -y = \frac{2}{3} \Leftrightarrow y = -\frac{2}{3}$

Por tanto, $\log_{\frac{1}{3}} \sqrt[3]{9} = -\frac{2}{3}$

f) $\log_{2\sqrt{2}} 0{,}25 = y \Leftrightarrow (2\sqrt{2})^y = 0{,}25 \Leftrightarrow \left(2 \cdot 2^{\frac{1}{2}}\right)^y = \frac{25}{100} \Leftrightarrow \left(2^{\frac{3}{2}}\right)^y = \frac{1}{4} \Leftrightarrow \left(2^{\frac{3}{2}}\right)^y = \frac{1}{2^2} \Leftrightarrow 2^{\frac{3y}{2}} = 2^{-2} \Leftrightarrow$

$\Leftrightarrow \frac{3y}{2} = -2 \Leftrightarrow 3y = -4 \Leftrightarrow y = -\frac{4}{3}$

Por tanto, $\log_{2\sqrt{2}} 0{,}25 = -\frac{4}{3}$

g) $\log_{\frac{1}{2}} \frac{1}{2\sqrt{8}} = y \Leftrightarrow \left(\frac{1}{2}\right)^y = \frac{1}{2\sqrt{8}} \Leftrightarrow 2^{-y} = \frac{1}{2\sqrt{2^3}} \Leftrightarrow 2^{-y} = \frac{1}{2 \cdot 2^{\frac{3}{2}}} \Leftrightarrow 2^{-y} = \frac{1}{2^{\frac{5}{2}}} \Leftrightarrow$

$\Leftrightarrow 2^{-y} = 2^{-\frac{5}{2}} \Leftrightarrow -y = -\frac{5}{2} \Leftrightarrow y = \frac{5}{2}$

Por tanto, $\log_{\frac{1}{2}} \frac{1}{2\sqrt{8}} = \frac{5}{2}$

h) $\log_{\frac{1}{2}} \sqrt[3]{16} = y \Leftrightarrow \left(\frac{1}{2}\right)^y = \sqrt[3]{16} \Leftrightarrow 2^{-y} = \sqrt[3]{2^4} \Leftrightarrow 2^{-y} = 2^{\frac{4}{3}} \Leftrightarrow -y = \frac{4}{3} \Leftrightarrow y = -\frac{4}{3}$

Por tanto, $\log_{\frac{1}{2}} \sqrt[3]{16} = -\frac{4}{3}$

i) $\ln \sqrt[5]{e^2} = y \Leftrightarrow e^y = \sqrt[5]{e^2} \Leftrightarrow e^y = e^{\frac{2}{5}} \Leftrightarrow y = \frac{2}{5}$

Por tanto, $\ln \sqrt[5]{e^2} = \frac{2}{5}$

j) $\ln \frac{e^2}{\sqrt{e}} = y \Leftrightarrow e^y = \frac{e^2}{\sqrt{e}} \Leftrightarrow e^y = \frac{e^2}{e^{\frac{1}{2}}} \Leftrightarrow e^y = e^{\frac{3}{2}} \Leftrightarrow y = \frac{3}{2}$

Por tanto, $\ln \frac{e^2}{\sqrt{e}} = \frac{3}{2}$

k) $\log 0{,}0001 = y \Leftrightarrow 10^y = 0{,}0001 \Leftrightarrow 10^y = 10^{-4} \Leftrightarrow y = -4$

Por tanto, $\log 0{,}0001 = -4$

l) $\log 0 = \text{no existe } (\log_a x \ \text{ existe } \Leftrightarrow x > 0)$

m) $\log(-10)^6 = y \Leftrightarrow 10^y = (-10)^6 \Leftrightarrow 10^y = 10^6 \Leftrightarrow y = 6$

Por tanto, $\log(-10)^6 = 6$

n) $\log(-10^6) = \text{no existe } (\log_a x \ \text{ existe } \Leftrightarrow x > 0)$

o) $\log_5 5\sqrt{5} = y \Leftrightarrow 5^y = 5\sqrt{5} \Leftrightarrow 5^y = 5 \cdot 5^{\frac{1}{2}} \Leftrightarrow 5^y = 5^{\frac{3}{2}} \Leftrightarrow y = \frac{3}{2}$

Por tanto, $\log_5 5\sqrt{5} = \frac{3}{2}$

p) $\log \sqrt{0'01} = y \Leftrightarrow 10^y = \sqrt{10^{-2}} \Leftrightarrow 10^y = 10^{-1} \Leftrightarrow y = -1$

Por tanto, $\log \sqrt{0'01} = -1$

q) $\log_6 \sqrt[5]{216^{-1}} = y \Leftrightarrow 6^y = \sqrt[5]{216^{-1}} \Leftrightarrow 6^y = \sqrt[5]{(6^3)^{-1}} \Leftrightarrow 6^y = \sqrt[5]{6^{-3}} \Leftrightarrow 6^y = 6^{-\frac{3}{5}} \Leftrightarrow y = -\frac{3}{5}$

Por tanto, $\log_6 \sqrt[5]{216^{-1}} = -\frac{3}{5}$

r) $\log_{\sqrt{\frac{1}{5}}} 0{,}04 = y \Leftrightarrow \left(\sqrt{\frac{1}{5}}\right)^y = 0{,}04 \Leftrightarrow \left(\sqrt{5^{-1}}\right)^y = \frac{4}{100} \Leftrightarrow \left(5^{-\frac{1}{2}}\right)^y = \frac{1}{25} \Leftrightarrow 5^{-\frac{y}{2}} = 5^{-2} \Leftrightarrow$

$\Leftrightarrow -\frac{y}{2} = -2 \Leftrightarrow -y = -4 \Leftrightarrow y = 4$

Por tanto, $\log_{\sqrt{\frac{1}{5}}} 0{,}04 = 4$

s) $\log_4 \frac{1}{\sqrt[3]{1024}} = y \Leftrightarrow 4^y = \frac{1}{\sqrt[3]{1024}} \Leftrightarrow (2^2)^y = \frac{1}{\sqrt[3]{2^{10}}} \Leftrightarrow 2^{2y} = \frac{1}{2^{\frac{10}{3}}} \Leftrightarrow 2^{2y} = 2^{-\frac{10}{3}} \Leftrightarrow$

$\Leftrightarrow 2y = -\frac{10}{3} \Leftrightarrow y = -\frac{5}{3}$

Por tanto, $\log_4 \frac{1}{\sqrt[3]{1024}} = -\frac{5}{3}$

t) $\log_{128} \sqrt[3]{2} = y \Leftrightarrow 128^y = \sqrt[3]{2} \Leftrightarrow (2^7)^y = 2^{\frac{1}{3}} \Leftrightarrow 2^{7y} = 2^{\frac{1}{3}} \Leftrightarrow 7y = \frac{1}{3} \Leftrightarrow y = \frac{1}{21}$

Por tanto, $\log_{128} \sqrt[3]{2} = \frac{1}{21}$

u) $\log_{\frac{1}{9}} \frac{\sqrt[4]{3}}{9} = y \Leftrightarrow \left(\frac{1}{9}\right)^y = \frac{\sqrt[4]{3}}{9} \Leftrightarrow \left(3^{-2}\right)^y = \frac{3^{\frac{1}{4}}}{3^2} \Leftrightarrow 3^{-2y} = 3^{-\frac{7}{4}} \Leftrightarrow -2y = -\frac{7}{4} \Leftrightarrow y = \frac{7}{8}$

Por tanto, $\log_{\frac{1}{9}} \frac{\sqrt[4]{3}}{9} = \frac{7}{8}$

v) $\log_3 \frac{\sqrt[4]{3}}{\sqrt{27}} = y \Leftrightarrow 3^y = \frac{\sqrt[4]{3}}{\sqrt{27}} \Leftrightarrow 3^y = \frac{3^{\frac{1}{4}}}{3^{\frac{3}{2}}} \Leftrightarrow 3^y = 3^{\frac{1}{4}-\frac{3}{2}} \Leftrightarrow 3^y = 3^{-\frac{5}{4}} \Leftrightarrow y = -\frac{5}{4}$

Por tanto, $\log_3 \frac{\sqrt[4]{3}}{\sqrt{27}} = -\frac{5}{4}$

w) $\log_2(-16) = \text{no existe}$ ($\log_a x$ existe $\Leftrightarrow x > 0$)

x) $\ln \frac{1}{e^3} = y \Leftrightarrow e^y = \frac{1}{e^3} \Leftrightarrow e^y = e^{-3} \Leftrightarrow y = -3$

Por tanto, $\ln \frac{1}{e^3} = -3$

y) $\log_{-3} 81 = \text{no existe}$, la base de un logaritmo debe ser un número real positivo y distinto de 1

z) $\log_a 1 = 0 \;\; \forall a > 0, a \neq 1$

2.- Halla el valor de las siguientes expresiones:

a) $\log_{25}\frac{1}{\sqrt[5]{5}}-\log_3 243+\log_{16}\frac{1}{4}\underset{(*)}{=}-\frac{1}{10}-5+\left(-\frac{1}{2}\right)=-\frac{1}{10}-5-\frac{1}{2}=\frac{-1-50-5}{10}=-\frac{56}{10}=-\frac{28}{5}$

$(*)\log_{25}\frac{1}{\sqrt[5]{5}}=y\Leftrightarrow 25^y=\frac{1}{\sqrt[5]{5}}\Leftrightarrow (5^2)^y=\frac{1}{5^{\frac{1}{5}}}\Leftrightarrow 5^{2y}=5^{-\frac{1}{5}}\Leftrightarrow 2y=-\frac{1}{5}\Leftrightarrow y=-\frac{1}{10}$

$(*)\log_3 243=y\Leftrightarrow {}^y=243\Leftrightarrow 3^y=3^5\Leftrightarrow y=5$

$(*)\log_{16}\frac{1}{4}=y\Leftrightarrow 16^y=\frac{1}{4}\Leftrightarrow (2^4)^y=2^{-2}\Leftrightarrow 2^{4y}=2^{-2}\Leftrightarrow 4y=-2\Leftrightarrow y=-\frac{2}{4}\Leftrightarrow y=-\frac{1}{2}$

b) $\log_2\sqrt[6]{0{,}5}-\log_{49}\frac{1}{7}-\log_{216}6-\log_4 64\underset{(*)}{=}-\frac{1}{6}-\left(-\frac{1}{2}\right)-\frac{1}{3}-3=-\frac{1}{6}+\frac{1}{2}-\frac{1}{3}-3=$

$=\frac{-1+3-2-18}{6}=-\frac{18}{6}=-3$

$(*)\log_2\sqrt[6]{0{,}5}=y\Leftrightarrow 2^y=\sqrt[6]{0{,}5}\Leftrightarrow 2^y=\sqrt[6]{\frac{1}{2}}\Leftrightarrow 2^y=\sqrt[6]{2^{-1}}\Leftrightarrow 2^y=2^{-\frac{1}{6}}\Leftrightarrow y=-\frac{1}{6}$

$(*)\log_{49}\frac{1}{7}=y\Leftrightarrow 49^y=\frac{1}{7}\Leftrightarrow (7^2)^y=7^{-1}\Leftrightarrow 7^{2y}=7^{-1}\Leftrightarrow 2y=-1\Leftrightarrow y=-\frac{1}{2}$

$(*)\log_{216}6=y\Leftrightarrow 216^y=6\Leftrightarrow (6^3)^y=6\Leftrightarrow 6^{3y}=6^1\Leftrightarrow 3y=1\Leftrightarrow y=\frac{1}{3}$

$(*)\log_4 64=y\Leftrightarrow 4^y=64\Leftrightarrow 4^y=4^3\Leftrightarrow y=3$

c) $\log_5(25^5\cdot 0{,}008^2)=y\Leftrightarrow 5^y=(25^5\cdot 0{,}008^2)\Leftrightarrow 5^y=(5^2)^5\cdot\left(\frac{8}{1000}\right)^2\Leftrightarrow 5^y=5^{10}\cdot\left(\frac{1}{125}\right)^2\Leftrightarrow$

$\Leftrightarrow 5^y=5^{10}\cdot(5^{-3})^2\Leftrightarrow 5^y=5^{10}\cdot 5^{-6}\Leftrightarrow 5^y=5^4\Leftrightarrow y=4$

Por tanto, $\log_5(25^5\cdot 0{,}008^2)=4$

<u>Otra forma (aplicando propiedades)</u>

$\log_5(25^5\cdot 0{,}008^2)\underset{\text{Prop.1}}{=}\log_5 25^5+\log_5 0{,}008^2=\log_5(5^2)^5+\log_5\left(\frac{8}{1000}\right)^2=\log_5 5^{10}+\log_5\left(\frac{1}{125}\right)^2=$

$=\log_5 5^{10}+\log_5(5^{-3})^2=\log_5 5^{10}+\log_5 5^{-6}\underset{\text{Prop.3}}{=}10\cdot\log_5 5-6\cdot\log_5 5=4\cdot\log_5 5\underset{\log_a a=1}{=}4\cdot 1=4$

d) $\log_2\left(\frac{4\cdot 0{,}125^{\frac{3}{2}}}{\sqrt{2}}\right)=y\Leftrightarrow 2^y=\left(\frac{4\cdot 0{,}125^{\frac{3}{2}}}{\sqrt{2}}\right)\Leftrightarrow 2^y=\left(\frac{2^2\cdot\left(\frac{125}{1000}\right)^{\frac{3}{2}}}{2^{\frac{1}{2}}}\right)\Leftrightarrow 2^y=\left(\frac{2^2\cdot\left(\frac{1}{8}\right)^{\frac{3}{2}}}{2^{\frac{1}{2}}}\right)\Leftrightarrow$

$\Leftrightarrow 2^y=\left(\frac{2^2\cdot(2^{-3})^{\frac{3}{2}}}{2^{\frac{1}{2}}}\right)\Leftrightarrow 2^y=\left(\frac{2^2\cdot 2^{-\frac{9}{2}}}{2^{\frac{1}{2}}}\right)\Leftrightarrow 2^y=\left(\frac{2^{-\frac{5}{2}}}{2^{\frac{1}{2}}}\right)\Leftrightarrow 2^y=2^{-3}\Leftrightarrow y=-3$

Por tanto, $\log_2\left(\dfrac{4\cdot 0{,}125^{\frac{3}{2}}}{\sqrt{2}}\right)=-3$

Otra forma (aplicando propiedades)

$$\log_2\left(\frac{4\cdot 0{,}125^{\frac{3}{2}}}{\sqrt{2}}\right)\underset{\text{Prop. 2}}{=}\log_2\left(4\cdot 0{,}125^{\frac{3}{2}}\right)-\log_2\sqrt{2}\underset{\substack{4=2^2\\ 0{,}125=\frac{125}{1000}=\frac{1}{8}=\frac{1}{2^3}=2^{-3}\\ \sqrt{2}=2^{\frac{1}{2}}}}{=}\log_2\left(2^2\cdot(2^{-3})^{\frac{3}{2}}\right)-\log_2 2^{\frac{1}{2}}=$$

$$=\log_2\left(2^2\cdot 2^{-\frac{9}{2}}\right)-\log_2 2^{\frac{1}{2}}=\log_2 2^{-\frac{5}{2}}-\log_2 2^{\frac{1}{2}}=-\frac{5}{2}\cdot\log_2 2-\frac{1}{2}\cdot\log_2 2=-\frac{6}{2}\cdot\log_2 2=-3\cdot\log_2 2\underset{\log_a a=1}{=}$$

$$=-3\cdot 1=-3$$

e) $\log_2\sqrt[5]{\dfrac{16^2}{0{,}5\cdot\sqrt{2}}}=y\Leftrightarrow 2^y=\sqrt[5]{\dfrac{16^2}{0{,}5\cdot\sqrt{2}}}\Leftrightarrow 2^y=\sqrt[5]{\dfrac{(2^4)^2}{\frac{1}{2}\cdot 2^{\frac{1}{2}}}}\Leftrightarrow 2^y=\sqrt[5]{\dfrac{2^8}{2^{-1}\cdot 2^{\frac{1}{2}}}}\Leftrightarrow 2^y=\sqrt[5]{\dfrac{2^8}{2^{-\frac{1}{2}}}}\Leftrightarrow$

$\Leftrightarrow 2^y=\sqrt[5]{2^{\frac{17}{2}}}\Leftrightarrow 2^y=2^{\frac{17}{10}}\Leftrightarrow y=\dfrac{17}{10}$

Por tanto, $\log_2\sqrt[5]{\dfrac{16^2}{0{,}5\cdot\sqrt{2}}}=\dfrac{17}{10}$

Otra forma (aplicando propiedades)

$$\log_2\sqrt[5]{\frac{16^2}{0{,}5\cdot\sqrt{2}}}=\log_2\left(\frac{(2^4)^2}{\frac{1}{2}\cdot 2^{\frac{1}{2}}}\right)^{\frac{1}{5}}=\log_2\left(\frac{2^8}{2^{-1}\cdot 2^{\frac{1}{2}}}\right)^{\frac{1}{5}}=\log_2\left(\frac{2^8}{2^{-\frac{1}{2}}}\right)^{\frac{1}{5}}=\log_2\left(2^{\frac{17}{2}}\right)^{\frac{1}{5}}=\log_2 2^{\frac{17}{10}}\underset{\text{Prop. 3}}{=}$$

$$=\frac{17}{10}\cdot\log_2 2\underset{\log_a a=1}{=}\frac{17}{10}\cdot 1=\frac{17}{10}$$

3.- Halla el valor de x en cada caso:

En todos los apartados aplicamos la definición de logaritmo y luego desarrollamos

a) $\log_x 7 = -2 \Leftrightarrow x^{-2} = 7 \Leftrightarrow \frac{1}{x^2} = 7 \Leftrightarrow 1 = 7x^2 \Leftrightarrow x^2 = \frac{1}{7} \Leftrightarrow x = \sqrt{\frac{1}{7}} \Leftrightarrow x = \frac{1}{\sqrt{7}} \underset{racionaliar}{\Leftrightarrow} x = \frac{\sqrt{7}}{7}$

b) $\log_x 7 = \frac{1}{2} \Leftrightarrow x^{\frac{1}{2}} = 7 \Leftrightarrow \sqrt{x} = 7 \Leftrightarrow (\sqrt{x})^2 = 7^2 \Leftrightarrow x = 49$

c) $\log_7 x^4 = 2 \Leftrightarrow 7^2 = x^4 \Leftrightarrow x = \pm\sqrt[4]{7^2} \underset{simplificar}{\Leftrightarrow} x = \pm\sqrt{7}$

d) $\log_x\left(\frac{1}{49}\right) = \frac{1}{4} \Leftrightarrow x^{\frac{1}{4}} = \frac{1}{49} \Leftrightarrow \sqrt[4]{x} = \frac{1}{49} \Leftrightarrow (\sqrt[4]{x})^4 = \left(\frac{1}{7^2}\right)^4 \Leftrightarrow x = \frac{1}{7^8} \Leftrightarrow x = 7^{-8}$

e) $\log_2 x = -\frac{1}{2} \Leftrightarrow 2^{-\frac{1}{2}} = x \Leftrightarrow x = \frac{1}{2^{\frac{1}{2}}} \Leftrightarrow x = \frac{1}{\sqrt{2}} \underset{racionalizar}{\Leftrightarrow} x = \frac{\sqrt{2}}{2}$

f) $\log_{\frac{1}{8}} x = \frac{1}{3} \Leftrightarrow \left(\frac{1}{8}\right)^{\frac{1}{3}} = x \Leftrightarrow x = \sqrt[3]{\frac{1}{8}} \Leftrightarrow x = \frac{1}{2}$

g) $\log_7 (7x) = 2 \Leftrightarrow 7^2 = 7x \Leftrightarrow x = \frac{7^2}{7} \Leftrightarrow x = 7$

h) $\log_x \frac{1}{3} = -\frac{1}{2} \Leftrightarrow x^{-\frac{1}{2}} = \frac{1}{3} \Leftrightarrow \frac{1}{x^{\frac{1}{2}}} = \frac{1}{3} \Leftrightarrow x^{\frac{1}{2}} = 3 \Leftrightarrow \sqrt{x} = 3 \Leftrightarrow x = 9$

i) $\log_x 0{,}001 = -3 \Leftrightarrow x^{-3} = 0{,}001 \Leftrightarrow \frac{1}{x^3} = \frac{1}{1000} \Leftrightarrow x^3 = 1000 \Leftrightarrow x = \sqrt[3]{1000} \Leftrightarrow x = 10$

j) $\log_x 27 = -\frac{1}{3} \Leftrightarrow x^{-\frac{1}{3}} = 27 \Leftrightarrow \frac{1}{x^{\frac{1}{3}}} = 27 \Leftrightarrow x^{\frac{1}{3}} = \frac{1}{27} \Leftrightarrow \sqrt[3]{x} = 3^{-3} \Leftrightarrow (\sqrt[3]{x})^3 = (3^{-3})^3 \Leftrightarrow$

$\Leftrightarrow x = 3^{-9} \Leftrightarrow x = \frac{1}{3^9} \Leftrightarrow x = \frac{1}{19683}$

k) $\log_x e = -3 \Leftrightarrow x^{-3} = e \Leftrightarrow \frac{1}{x^3} = e \Leftrightarrow x^3 = \frac{1}{e} \Leftrightarrow x = \sqrt[3]{\frac{1}{e}} \Leftrightarrow x = \frac{1}{\sqrt[3]{e}}$

l) $\log_x 0{,}015625 = -3 \Leftrightarrow x^{-3} = 0{,}015625 \Leftrightarrow \dfrac{1}{x^3} = \dfrac{15625}{1000000} \Leftrightarrow \dfrac{1}{x^3} = \dfrac{1}{64} \Leftrightarrow x^3 = 64 \Leftrightarrow x = 4$

4.- Sabiendo que $\log 2 = 0{,}301$ **y** $\log 3 = 0{,}477$ **calcula:**

a) $\log 12 = \log(2^2 \cdot 3) \underset{\text{Prop.1}}{=} \log 2^2 + \log 3 \underset{\text{Prop.3}}{=} 2\log 2 + \log 3 = 2\cdot(0{,}301) + 0{,}477 = 0{,}602 + 0{,}477 = 1{,}079$

b) $\log 0{,}0002 = \log\left(\dfrac{2}{10000}\right) \underset{\text{Prop.2}}{=} \log 2 - \log 10000 = \log 2 - \log 10^4 \underset{\text{Prop.3}}{=} \log 2 - 4\log 10 \underset{\log_a a = 1}{=}$

$= 0{,}301 - 4\cdot 1 = 0{,}301 - 4 = -3{,}699$

c) $\log \sqrt[5]{6} = \log 6^{\frac{1}{5}} \underset{\text{Prop.3}}{=} \dfrac{1}{5}\log 6 = \dfrac{1}{5}\log(2\cdot 3) \underset{\text{Prop.1}}{=} \dfrac{1}{5}(\log 2 + \log 3) = \dfrac{1}{5}(0{,}301 + 0{,}477) = 0{,}1556$

d) $\log 27000 = \log(27\cdot 1000) = \log(3^3 \cdot 10^3) \underset{\text{Prop.1}}{=} \log 3^3 + \log 10^3 \underset{\text{Prop.3}}{=} 3\log 3 + 3\log 10 =$

$3\cdot 0{,}477 + 3\cdot 1 = 1{,}431 + 3 = 4{,}431$

e) $\log \dfrac{\sqrt{32}}{6} \underset{\text{Prop.2}}{=} \log\sqrt{32} - \log 6 = \log\sqrt{2^5} - \log(2\cdot 3) \underset{\text{Prop.1}}{=} \log 2^{\frac{5}{2}} - (\log 2 + \log 3) \underset{\substack{\text{Prop.3} \\ \text{Quitar paréntesis}}}{=}$

$= \dfrac{5}{2}\log 2 - \log 2 - \log 3 = \underset{\substack{\uparrow \\ \frac{5}{2}-1=\frac{3}{2}}}{=} \dfrac{3}{2}\log 2 - \log 3 = \dfrac{3}{2}\cdot 0{,}301 - 0{,}477 = -0{,}0255$

f) $\log 0{,}0125 = \log\left(\dfrac{125}{10000}\right) \underset{\text{simplificar}}{=} \log\left(\dfrac{1}{80}\right) \underset{\text{Prop.2}}{=} \log 1 - \log 80 = 0 - \log(8\cdot 10) = -\log(2^3\cdot 10) \underset{\text{Prop.1}}{=}$

$= -(\log 2^3 + \log 10) \underset{\substack{\text{Quitar paréntesis y} \\ \text{Prop.3}}}{=} = -3\log 2 - \log 10 = -3\cdot 0{,}301 - 1 = -1{,}903$

g) $\log\sqrt[5]{0{,}48} = \log\sqrt[5]{\dfrac{48}{100}} = \log\left(\dfrac{2^4\cdot 3}{10^2}\right)^{\frac{1}{5}} = \log\left(\dfrac{2^{\frac{4}{5}}\cdot 3^{\frac{1}{5}}}{10^{\frac{2}{5}}}\right) \underset{\text{Prop.2}}{=} \log\left(2^{\frac{4}{5}}\cdot 3^{\frac{1}{5}}\right) - \log 10^{\frac{2}{5}} \underset{\text{Prop.1}}{=}$

$= \log 2^{\frac{4}{5}} + \log 3^{\frac{1}{5}} - \log 10^{\frac{2}{5}} \underset{\text{Prop.3}}{=} = \dfrac{4}{5}\log 2 + \dfrac{1}{5}\log 3 - \dfrac{2}{5}\log 10 = \dfrac{4}{5}\cdot 0{,}301 + \dfrac{1}{5}\cdot 0{,}477 - \dfrac{2}{5}\cdot 1 =$

$= 0{,}2408 + 0{,}0954 - 0{,}4 = -0{,}0638$

h) $\log\dfrac{1}{\sqrt[4]{0{,}6}} \underset{\text{Prop.2}}{=} \log 1 - \log\sqrt[4]{0{,}6} = 0 - \log\sqrt[4]{0{,}6} = -\log\sqrt[4]{\dfrac{6}{10}} = -\log\sqrt[4]{\dfrac{2\cdot 3}{10}} = -\log\left(\dfrac{2\cdot 3}{10}\right)^{\frac{1}{4}} \underset{\text{Prop.3}}{=}$

$$= -\frac{1}{4}\cdot\log\left(\frac{2\cdot 3}{10}\right) \underset{\substack{\text{Prop.2}\\ \text{Prop.1}}}{=} -\frac{1}{4}\cdot(\log 2+\log 3-\log 10) = -\frac{1}{4}\cdot(0{,}301+0{,}477-1) = -\frac{1}{4}\cdot(-0{,}222) = 0{,}0555$$

i) $\log 3{,}6 = \log\left(\frac{36}{10}\right) = \log\left(\frac{2^2\cdot 3^2}{10}\right) \underset{\substack{\text{Prop.2}\\ \text{Prop.1}}}{=} \log 2^2 + \log 3^2 - \log 10 \underset{\text{Prop.3}}{=} 2\log 2 + 2\log 3 - 1 =$

$= 2\cdot 0{,}301 + 2\cdot 0{,}477 - 1 = 0{,}556$

j) $\log 360 = \log(36\cdot 10) \underset{\text{Prop.1}}{=} \log 36 + \log 10 = \log(2^2\cdot 3^2) + 1 \underset{\text{Prop.1}}{=} \log 2^2 + \log 3^2 + 1 \underset{\text{Prop.3}}{=}$

$= 2\log 2 + 2\log 3 + 1 == 2\cdot 0{,}301 + 2\cdot 0{,}477 + 1 = 2{,}556$

k) $\log(5\cdot\sqrt[3]{9}) \underset{\text{Prop.1}}{=} \log 5 + \log\sqrt[3]{9} = \log\left(\frac{10}{2}\right) + \log\sqrt[3]{3^2} \underset{\text{Prop.2}}{=} \log 10 - \log 2 + \log 3^{\frac{2}{3}} \underset{\text{Prop.3}}{=}$

$= \log 10 - \log 2 + \frac{2}{3}\log 3 = 1 - 0{,}301 + \frac{2}{3}\cdot 0{,}477 = 1{,}017$

l) $\log(3{,}2\cdot 2{,}7^3) \underset{\text{Prop.1}}{=} \log 3{,}2 + \log 2{,}7^3 \underset{\text{Prop.3}}{=} \log 3{,}2 + 3\log 2{,}7 = \log\left(\frac{32}{10}\right) + 3\log\left(\frac{27}{10}\right) =$

$= \log\left(\frac{2^5}{10}\right) + 3\log\left(\frac{3^3}{10}\right) \underset{\text{Prop.2}}{=} \log 2^5 - \log 10 + 3(\log 3^3 - \log 10) \underset{\text{Prop.3}}{=} 5\log 2 - \log 10 + 3(3\log 3 - \log 10) =$

$\underset{\substack{\text{quitar}\\ \text{paréntesis}}}{=} 5\log 2 - \log 10 + 9\log 3 - 3\log 10 = 5\log 2 + 9\log 3 - 4\log 10 = 5\cdot 0{,}301 + 9\cdot 0{,}477 - 4\cdot 1 = 1{,}798$

5.- Pasa a forma algebraica:

a) $\frac{1}{2}\log C = 3\log A - \log 2 + 2\log B$

$\log C^{\frac{1}{2}} = \log A^3 - \log 2 + \log B^2$

$\log\sqrt{C} = \log\left(\frac{A^3}{2}\right) + \log B^2$

$\log\sqrt{C} = \log\left(\frac{A^3}{2}\cdot B^2\right)$

$\sqrt{C} = \frac{A^3\cdot B^2}{2}$

b) $\frac{1}{3}\log z = \frac{2}{3}\log x - \log y + 3\log s$

$\log z^{\frac{1}{3}} = \log x^{\frac{2}{3}} - \log y + \log s^3$

$$\log\sqrt[3]{z}=\log\left(\frac{\sqrt[3]{x^2}}{y}\right)+\log s^3$$

$$\log\sqrt[3]{z}=\log\left(\frac{\sqrt[3]{x^2}}{y}\cdot s^3\right)$$

$$\sqrt[3]{z}=\frac{\sqrt[3]{x^2}\cdot s^3}{y}$$

$$z=\left(\frac{\sqrt[3]{x^2}\cdot s^3}{y}\right)^3$$

$$z=\frac{x^2\cdot s^9}{y^3}$$

c) $2-\log D=2\log A-3\log B-4\log C$

$$\log 100-\log D=\log A^2-\log B^3-\log C^4$$

$$\log\left(\frac{100}{D}\right)=\log\left(\frac{A^2}{B^3}\right)-\log C^4$$

$$\log\left(\frac{100}{D}\right)=\log\left(\frac{A^2}{B^3}:C^4\right)$$

$$\log\left(\frac{100}{D}\right)=\log\left(\frac{A^2}{B^3\cdot C^4}\right)$$

$$\frac{100}{D}=\frac{A^2}{B^3\cdot C^4}$$

$$100=\frac{A^2\cdot D}{B^3\cdot C^4}$$

d) $\log A=\frac{1}{2}-\frac{1}{3}\log B+\log C-\frac{2}{5}\log D$

$$\log A=\log 10^{\frac{1}{2}}-\log B^{\frac{1}{3}}+\log C-\log D^{\frac{2}{5}}$$

$$\log A=\log\sqrt{10}-\log\sqrt[3]{B}+\log C-\log\sqrt[5]{D^2}$$

$$\log A=\log\left(\frac{\sqrt{10}}{\sqrt[3]{B}}\right)+\log C-\log\sqrt[5]{D^2}$$

$$\log A=\log\left(\frac{\sqrt{10}}{\sqrt[3]{B}}\cdot C\right)-\log\sqrt[5]{D^2}$$

$$\log A=\log\left(\frac{\sqrt{10}\cdot C}{\sqrt[3]{B}}:\sqrt[5]{D^2}\right)$$

$$\log A = \log\left(\frac{\sqrt{10}\cdot C}{\sqrt[3]{B}\cdot\sqrt[5]{D^2}}\right) \Rightarrow A = \frac{\sqrt{10}\cdot C}{\sqrt[3]{B}\cdot\sqrt[5]{D^2}}$$

6.- Toma logaritmos en las siguientes expresiones y desarrolla:

a) $A = \frac{x^3\cdot y}{z^5} \Rightarrow \log A = \log\left(\frac{x^3\cdot y}{z^5}\right) \underset{\text{Prop.2}}{\Rightarrow} \log A = \log(x^3\cdot y) - \log z^5 \underset{\text{Prop.1}}{\Rightarrow} \log A = \log x^3 + \log y - \log z^5 \underset{\text{Prop.3}}{\Rightarrow}$

$\Rightarrow \log A = 3\log x + \log y - 5\log z$

b) $B = \sqrt{x^3\cdot y^5\cdot z^2} \Rightarrow \log B = \log\sqrt{x^3\cdot y^5\cdot z^2} \Rightarrow \log B = \log\left(x^{\frac{3}{2}}\cdot y^{\frac{5}{2}}\cdot z\right) \underset{\text{Prop.1}}{\Rightarrow} \log B = \log x^{\frac{3}{2}} + \log y^{\frac{5}{2}} + \log z \underset{\text{Prop.3}}{\Rightarrow}$

$\Rightarrow \log B = \frac{3}{2}\log x + \frac{5}{2}\log y + \log z$

c) $C = \frac{X^2}{D\cdot\sqrt{A}} \Rightarrow \log C = \log\left(\frac{X^2}{D\cdot\sqrt{A}}\right) \underset{\text{Prop.2}}{\Rightarrow} \log C = \log X^2 - \log(D\cdot\sqrt{A}) \underset{\text{Prop.1}}{\Rightarrow} \log C = \log X^2 - \left(\log D + \log A^{\frac{1}{2}}\right) \underset{\text{Prop.3}}{\Rightarrow}$

$\Rightarrow \log C = 2\log X - \left(\log D + \frac{1}{2}\log A\right) \underset{\text{quitar paréntess}}{\Rightarrow} \log C = 2\log X - \log D - \frac{1}{2}\log A$

d) $D = \frac{A^5\cdot\sqrt{B}}{C^4} \Rightarrow \log D = \log\left(\frac{A^5\cdot\sqrt{B}}{C^4}\right) \underset{\text{Prop.2}}{\Rightarrow} \log D = \log(A^5\cdot\sqrt{B}) - \log C^4 \underset{\text{Prop.1}}{\Rightarrow} \log D = \log A^5 + \log B^{\frac{1}{2}} - \log C^4 \underset{\text{Prop.3}}{\Rightarrow}$

$\Rightarrow \log D = 5\log A + \frac{1}{2}\log B - 4\log C$

e) $E = \sqrt{\frac{A}{B\cdot\sqrt{C}}} \Rightarrow \log E = \log\sqrt{\frac{A}{B\cdot\sqrt{C}}} \Rightarrow \log E = \log\sqrt{\frac{A}{B\cdot C^{\frac{1}{2}}}} \Rightarrow \log E = \log\left(\frac{A^{\frac{1}{2}}}{B^{\frac{1}{2}}\cdot C^{\frac{1}{4}}}\right) \underset{\text{Prop.2}}{\Rightarrow}$

$\Rightarrow \log E = \log A^{\frac{1}{2}} - \log\left(B^{\frac{1}{2}}\cdot C^{\frac{1}{4}}\right) \underset{\text{Prop.1}}{\Rightarrow} \log E = \log A^{\frac{1}{2}} - \left(\log B^{\frac{1}{2}} + \log C^{\frac{1}{4}}\right) \underset{\text{Prop.3 Quitar paréntesis}}{\Rightarrow}$

$\Rightarrow \log E = \frac{1}{2}\log A - \frac{1}{2}\log B - \frac{1}{4}\log C$

f) $F = \sqrt[3]{\frac{A^2}{B\cdot\sqrt{C}}} \Rightarrow \log F = \log\sqrt[3]{\frac{A^2}{B\cdot\sqrt{C}}} \Rightarrow \log F = \log\sqrt[3]{\frac{A^2}{B\cdot C^{\frac{1}{2}}}} \Rightarrow \log F = \log\left(\frac{A^{\frac{2}{3}}}{B^{\frac{1}{3}}\cdot C^{\frac{1}{6}}}\right) \underset{\text{Prop.2}}{\Rightarrow}$

$\Rightarrow \log F = \log A^{\frac{2}{3}} - \log\left(B^{\frac{1}{3}}\cdot C^{\frac{1}{6}}\right) \underset{\text{Prop.1}}{\Rightarrow} \log F = \log A^{\frac{2}{3}} - \left(\log B^{\frac{1}{3}} + \log C^{\frac{1}{6}}\right) \underset{\text{Prop.3 Quitar paréntesis}}{\Rightarrow}$

$\Rightarrow \log F = \frac{2}{3}\log A - \frac{1}{3}\log B - \frac{1}{6}\log C$

7.- Sabiendo que $\log 2 = 0{,}301$**,** $\log 3 = 0{,}477$ **y utilizando el cambio de base calcula:**

$$\text{CAMBIO DE BASE} \rightarrow \log_a x = \frac{\log_b x}{\log_b a}$$

a) $\log_3 32 = \frac{\log 32}{\log 3} = \frac{\log 2^5}{\log 3} = \frac{5\log 2}{\log 3} = \frac{5 \cdot 0{,}301}{0{,}477} = 3{,}155$

b) $\log_4 0{,}3 = \frac{\log 0{,}3}{\log 4} = \frac{\log\left(\frac{3}{10}\right)}{\log 2^2} = \frac{\log 3 - \log 10}{2\log 2} = \frac{0{,}477 - 1}{2 \cdot 0{,}301} = \frac{-0{,}523}{0{,}602} = -0{,}869$

c) $\log_{\sqrt{2}} 27 = \frac{\log 27}{\log \sqrt{2}} = \frac{\log 3^3}{\log 2^{\frac{1}{2}}} = \frac{3\log 3}{\frac{1}{2}\log 2} = \frac{3 \cdot 0{,}477}{\frac{1}{2} \cdot 0{,}301} = \frac{1{,}431}{0{,}1505} = 9{,}508$

d) $\log_8 2 = \frac{\log 2}{\log 8} = \frac{\log 2}{\log 2^3} = \frac{\log 2}{3\log 2} = \frac{1}{3}$

e) $\log_{\sqrt{3}} 8 = \frac{\log 8}{\log \sqrt{3}} = \frac{\log 2^3}{\log 3^{\frac{1}{2}}} = \frac{3\log 2}{\frac{1}{2}\log 3} = \frac{3 \cdot 0{,}301}{\frac{1}{2} \cdot 0{,}477} = \frac{0{,}903}{0{,}2385} = 3{,}786$

f) $\log_{0{,}5} \sqrt[5]{3} = \frac{\log \sqrt[5]{3}}{\log 0{,}5} = \frac{\log 3^{\frac{1}{5}}}{\log \frac{1}{2}} = \frac{\frac{1}{5} \cdot \log 3}{\log 1 - \log 2} = \frac{\frac{1}{5} \cdot 0{,}477}{0 - 0{,}301} = \frac{0{,}0954}{-0{,}301} = -0{,}317$

g) $\log_{\frac{1}{\sqrt{2}}} \sqrt[3]{0{,}03} = \frac{\log \sqrt[3]{0{,}03}}{\log \frac{1}{\sqrt{2}}} = \frac{\log \sqrt[3]{\frac{3}{100}}}{\log 2^{-\frac{1}{2}}} = \frac{\log\left(\frac{3}{100}\right)^{\frac{1}{3}}}{-\frac{1}{2}\log 2} = \frac{\frac{1}{3}\log\left(\frac{3}{10^2}\right)}{-\frac{1}{2}\log 2} = \frac{\frac{1}{3} \cdot \left(\log 3 - \log 10^2\right)}{-\frac{1}{2}\log 2} =$

$= \frac{\frac{1}{3} \cdot (\log 3 - 2\log 10)}{-\frac{1}{2}\log 2} = \frac{\frac{1}{3} \cdot (0{,}477 - 2)}{-\frac{1}{2} \cdot 0{,}301} = \frac{\frac{1}{3} \cdot (-1{,}523)}{-0{,}1505} = 3{,}373$