SOLUCIONES

OPCIÓN A

1 a) $|A|=0 \Rightarrow \text{rgo } A \le 2$; $\begin{cases} m \neq 2 \Rightarrow \begin{vmatrix} -2 & 4 \\ -1 & m \end{vmatrix} \neq 0 \Rightarrow \text{rgo } A = 2 \\ m = 2 \Rightarrow \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix} \neq 0 \Rightarrow \text{rgo } A = 2 \end{cases}$ $\boxed{\text{Por tanto rgo } A = 2 \ \forall m}$

b) $|A^{20}| = |A|^{20} = 0^{20} = 0$

c) $\left(\begin{array}{ccc|c} -2 & 4 & 2 & 0 \\ -1 & -2 & -2 & 0 \\ -1 & 2 & 1 & 0 \end{array}\right) \underset{\substack{-2F_2+F_1 \\ -2F_3+F_1}}{\longleftrightarrow} \left(\begin{array}{ccc|c} -2 & 4 & 2 & 0 \\ 0 & 8 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \longleftrightarrow \left(\begin{array}{ccc|c} -1 & 2 & 1 & 0 \\ 0 & 4 & 3 & 0 \end{array}\right)$

Solución:

$$\boxed{\begin{array}{l} z = \lambda \\ y = -\frac{3}{4}\lambda \\ x = 2(-\frac{3}{4}\lambda) + \lambda = -\frac{\lambda}{2} \end{array}}$$

d) $\left(\begin{array}{ccc|c} -2 & 4 & 2 & -2 \\ -1 & 0 & 0 & 0 \\ -1 & 2 & 1 & -1 \end{array}\right) \underset{\substack{-2F_2+F_1 \\ -2F_3+F_1}}{\longleftrightarrow} \left(\begin{array}{ccc|c} -2 & 4 & 2 & -2 \\ 0 & 4 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right) \underset{F_1-F_2}{\longleftrightarrow} \left(\begin{array}{ccc|c} -2 & 0 & 0 & 0 \\ 0 & 4 & 2 & -2 \end{array}\right) \longleftrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & -1 \end{array}\right)$

$$\boxed{\left.\begin{array}{l} x = 0 \\ y = \frac{-1-\lambda}{2} \\ z = \lambda \end{array}\right\} \text{Sol}}$$

2 a) $\text{Dom} f = \mathbb{R} \setminus \{-1, 1\}$

b) $f'(x) = \frac{4}{(x+1)^2} > 0 \ \forall x \in \text{Dom} f$ $\boxed{f \text{ creciente en } (-\infty,-1) \text{ y en } (-1,1) \text{ y en } (1,+\infty)}$

c) $\int_{-1/2}^{1/2} |f| = \int_{-1/2}^{1/2} \left|\frac{x-3}{x+1}\right| dx = \int_{-1/2}^{1/2} \left|1 - \frac{4}{x+1}\right| dx = \int_{-1/2}^{1/2} \left(\frac{4}{x+1} - 1\right) dx = 4\ln(x+1) - x \Big]_{-\frac{1}{2}}^{1/2} = 4\ln 3 - 1$

$1 - \frac{4}{x+1} \le 0$ en $[-1/2, 1/2]$

3 a) $(1+2\lambda) + \lambda = 1 \Leftrightarrow 3\lambda = 0 \Leftrightarrow \lambda = 0$ $\boxed{\text{Se cortan en } (1,0,0)}$

$\vec{v}_r = (2,1,1)$ $\vec{v}_s = (-1,1,1)$ $\left(\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -\vec{i} + \vec{k} + \vec{j} = (-1,1,1) \right)$

$0 = 2(-1) + 1\cdot 1 + 1\cdot 1 = \langle \vec{v}_r, \vec{v}_s \rangle = \|\vec{v}_r\| \cdot \|\vec{v}_s\| \cos(\underbrace{\widehat{\vec{v}_r, \vec{v}_s}}_{\alpha}) \Rightarrow \cos\alpha = 0 \Rightarrow \boxed{\alpha = \pi/2}$

b) $\vec{v}_r \times \vec{v}_s = (0,-3,3)$ $\left(\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \vec{i} - \vec{j} + 2\vec{k} + \vec{k} - \vec{i} - 2\vec{j} = (0,-3,3) \right)$

$$\boxed{\left.\begin{array}{l} x = 0 \\ y = -\lambda \\ z = \lambda \end{array}\right\}}$$

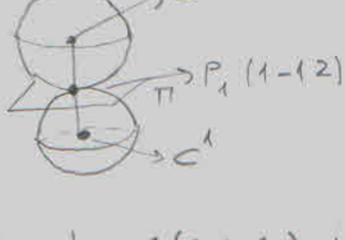
4 $\overrightarrow{P_1P_2} = (1,-2,-2)$ $\overrightarrow{P_1P_3} = (2,2,0)$ $\pi: \begin{vmatrix} x-1 & y+1 & z-2 \\ 1 & -2 & -2 \\ 2 & 2 & 0 \end{vmatrix} = 0$

a)

$$\boxed{\pi: 2x - 2y + 3z - 10 = 0}$$

b)

$$\boxed{\left.\begin{array}{l} x = 1 + 2\lambda \\ y = -1 - 2\lambda \\ z = 2 + 3\lambda \end{array}\right\}}$$



c) $r^2 = 17$

Ecuación esfera de centro $c(c_1 c_2 c_3)$ y radio $\sqrt{17}$: $\boxed{(x-c_1)^2 + (y-c_2)^2 + (z-c_3)^2 = 17}$

$(1-c_1, -1-c_2, 2-c_3) = \overrightarrow{cP_1} = \alpha(2,-2,3) \Rightarrow 17 = \|\overrightarrow{cP_1}\|^2 = \alpha^2(4+4+9) = 17\alpha^2 \Rightarrow \alpha^2 = 1$

$\Rightarrow \alpha = \pm 1 \Rightarrow c_1 = 1 - 2(\pm 1)$, $c_2 = -1 + 2(\pm 1)$ $c_3 = 2 - 3(\pm 1)$

$$\boxed{\begin{array}{l} c^1 = (1-2, -1+2, 2-3) = (-1,1,-1) \\ c^2 = (1+2, -1-2, 2+3) = (3,-3,5) \end{array}} \quad \boxed{\begin{array}{l} (x+1)^2 + (y-1)^2 + (z+1)^2 = 17 \\ (x-3)^2 + (y+3)^2 + (z-5)^2 = 17 \end{array}}$$

OPCIÓN B

(1) $v_r = (-2,1,-1)$ $\left(\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 1 & -1 & -3 \end{vmatrix} = -3\vec{i} - \vec{j} - \vec{k} - \vec{k} - \vec{i} + 3\vec{j} = (-4,2,-2) \right)$

a) $v_s = (0,0,1)$ $\left(\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \vec{k} = (0,0,1) \right)$

Como v_r y v_s son L.I y el sistema $\left\{ \begin{matrix} r \\ s \end{matrix} \right.$ es incompatible se deduce que $\boxed{r, s \text{ se cruzan}}$.

Sean $P(1,3,0) \in r$ y $Q(2,-3,0) \in s$; $\overrightarrow{PQ} = (1,-6,0)$

$$d(r,s) = \frac{\left| \begin{vmatrix} 1 & -6 & 0 \\ -2 & 1 & -1 \\ 0 & 0 & 1 \end{vmatrix} \right|}{\| v_r \times v_s \|} = \boxed{\frac{11}{\sqrt{5}}} \qquad \left(v_r \times v_s = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 1 & -1 \\ 0 & 0 & 1 \end{vmatrix} = \vec{i} + 2\vec{j} = (1,2,0) \right)$$

$\| v_r \times v_s \| = \sqrt{1^2+2^2} = \sqrt{5}$

b) Sea $R \in r$ $\overrightarrow{RP} = (1-(1+2\lambda), 2-(3-\lambda), -1-\lambda) = (-2\lambda, -1+\lambda, -1-\lambda)$ $\left(r: \begin{matrix} x = 1+2\lambda \\ y = 3-\lambda \\ z = \lambda \end{matrix} \right)$

$\overrightarrow{RP} \perp (-2,1,-1) \Leftrightarrow (-2\lambda)(-2) + (-1+\lambda)\cdot 1 + (-1-\lambda)\cdot(-1) = 0 \Leftrightarrow \dots \Leftrightarrow \lambda = 0$

Así $R(1,3,0) = \frac{P+P'}{2} = \frac{(1+x, 2+y, -1+z)}{2}$ $(P'(xyz)) \Rightarrow \boxed{P'(1,4,1)}$

c) $\underbrace{XY}_{\pi_1} : z = 0$

$\underbrace{YZ}_{\pi_2} : x = 0$

$$\frac{|\lambda|}{1} = d(\underbrace{(1+2\lambda, 3-\lambda, \lambda)}_{\text{pto genérico de } r}, \pi_1) = d((1+2\lambda, 3-\lambda, \lambda), \pi_2) = \frac{|1+2\lambda|}{1}$$

$\Leftrightarrow |\lambda| = |1+2\lambda| \Leftrightarrow \lambda = 1+2\lambda$ ó $\lambda = -1-2\lambda \Leftrightarrow \lambda = -1$ ó $\lambda = -1/3$ $\boxed{\begin{matrix} (-1,4,-1) \\ (1/3, 10/3, -1/3) \end{matrix}}$

(2) a)
$$\lim_{x\to 0} \frac{\sqrt{1+\operatorname{sen} x} - \sqrt{1-\operatorname{sen} x}}{x} \overset{(x \text{ conjugado})}{=} \lim_{x\to 0} \frac{(1+\operatorname{sen} x)-(1-\operatorname{sen} x)}{(\sqrt{1+\operatorname{sen} x} + \sqrt{1-\operatorname{sen} x})\, x} = \lim_{x\to 0} \frac{2 \operatorname{sen} x}{(\sqrt{1+\operatorname{sen} x} + \sqrt{1-\operatorname{sen} x})\, x} =$$

$$= \underbrace{\lim_{x\to 0} \frac{\operatorname{sen} x}{x}}_{1} \cdot \underbrace{\lim_{x\to 0} \frac{2}{(\sqrt{1+\operatorname{sen} x} + \sqrt{1-\operatorname{sen} x})}}_{1} = 1$$

b) $\int (3x+5)\cos x\, dx = (3x+5)\operatorname{sen} x - \int 3\operatorname{sen} x\, dx = \boxed{(3x+5)\operatorname{sen} x + 3\cos x + k}$

$\cos x = dv$ $v = \operatorname{sen} x$

$3x+5 = u$ $du = 3$

c) $f'(x) = \frac{e^x(1-x)}{x^2}$

$\left. \begin{matrix} f'(x) > 0 \Leftrightarrow x \in (-\infty, 1) \Rightarrow f \text{ creciente en } (-\infty,0) \text{ y } (0,1) \\ f'(x) < 0 \Leftrightarrow x \in (1,+\infty) \Rightarrow f \text{ decreciente en } (1,+\infty) \end{matrix} \right\} \overset{f'(1)=0}{\Rightarrow} x = 1$ máximo local

(3) a) Restando a la 1ª ec la 2ª queda $\left(\begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \right) Y = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ ó $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} Y = \begin{pmatrix} 1 & -2 \\ 1 & 2 \end{pmatrix}$

Como $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ resulta $Y = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 1 & 2 \end{pmatrix} = \boxed{\begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix} = Y}$ y $X = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix} = \boxed{\begin{pmatrix} -1 & 3 \\ -5 & -1 \end{pmatrix} = X}$

b) $Z^2 \cdot 3 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} Z^{-1} = 3 \cdot Z^2 \cdot Z^{-1} = 3Z(Z \cdot Z^{-1}) = 3Z$; así $3Z = \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix} \Rightarrow \boxed{Z = \begin{pmatrix} 1/3 & 1 \\ 1/3 & 2/3 \end{pmatrix}}$

(4) a) $\begin{pmatrix} m & 1 \\ 1 & m \\ m & m \end{pmatrix}$ $\begin{matrix} F_1 \\ F_2 \end{matrix} \begin{vmatrix} m & 1 \\ 1 & m \end{vmatrix} = m^2 - 1 \Rightarrow m \neq 1, -1$ el sist es C.D

Pero además si $m = -1$ el menor $\begin{matrix} F_2 \\ F_3 \end{matrix} \begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix} = -2 \neq 0$.

$\boxed{\text{El sist es C.D } \forall m \neq 1}$ (solución trivial $(0,0)$)

$\boxed{\text{Para } m = 1 \text{ es C.I}}$ pues es equivalente a $x + y = 0$

b) $x = -\lambda$
$y = \lambda$ $\boxed{\text{Sol } \{(-\lambda, \lambda) / \lambda \in \mathbb{R}\}}$